SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

November 2010
Assessment Task 1
Year 11

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- All necessary working should be shown in every question if full marks are to be awarded.
- All answers to be given in simplified exact form unless otherwise stated.
- Marks may not be awarded for messy or badly arranged work

Total Marks – 80

- Attempt questions 1-5
- Start each new question in a separate answer booklet.
- Hand in your answers in 5 separate bundles:

  Question 1,
  Question 2,
  Question 3,
  Question 4 and
  Question 5

Examiner: *A Ward*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate

Start a new booklet.

Question 1 (16 marks). **Marks**

a) Determine whether the line $y = 3x + 7$ passes through the point (27, 86). 1

b) What is the coefficient of $11x^4$? 1

c) Rewrite $\log_5 625 = x$ in exponential form (but do not solve for *x*). 1

d) State whether or not $\{(1,2),\ (2,4),\ (1,5)\}$ is a function. 1

e) Without solving the equation $2x^2 - x - 3 = 0$, determine and justify whether the roots are: 3

(i) Real or non-real.

(ii) Equal or distinct.

(iii)Rational or irrational.

f) Determine whether the following functions are odd, even or neither: 3

(i) $f(x) = x^3 + 3x^2 + 7x$

(ii) $f(x) = \dfrac{2}{x^3}$

(iii) $f(x) = \dfrac{1}{x^5 + x^3}$

g) If the equation $x^2 + 3px + p$, where *p* is a non-zero constant , has equal roots; find the value of p. 2

h) The probabilities that Albert, Byron and Charlie will pass an examination are $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{4}$ respectively. What is the probability that Albert and Charlie will pass and Byron will fail? 2

i) Simplify first and find the value of x: 2

$$\log_{10} 5 + 2\log_{10} 8 - \frac{1}{2}\log_{10} 16 = \log_{10} x$$

End of Question 1

Start a new booklet.

Question 2 (16 Marks). **Marks**

a) A factory produces components of which 0.6% are defective. The components are packed in boxes of 10. A box is selected at random. Find the probability that the box contains exactly 2 defective components. 2

b) Differentiate the following with respect to *x*: 6

(i) $x^5 - 4x^4 + 2x^2 - 7$

(ii) $\left(3x+2\right)^7$

(iii) $x\left(2x^3 - 13\right)$

(iv) $\dfrac{8x^2+6}{x-1}; x \neq 1$

c) Find the equation of the tangent to the curve $y = x^2 + 2x$, that is parallel to the line $y = 4x + 1$. 3

d) A parabola has vertex (-2,-3) and directrix $y = -1$. Find the: 5

(i) focal length

(ii) focus

(iii) axis of symmetry

(iv) equation of the parabola

End of Question 2

Start a new booklet.

Question 3 (16 marks). **Marks**

a) Find the $\lim\limits_{x\to 2}\dfrac{2x^2-8}{x-2}$ 1

b) Find to 3 significant figures the value of x for which $5^x = 7$ 2

c) Sketch the regions satisfying the simultaneous inequations such that: 3

$$\left\{(x,y): y \geq x^2\right\} \cap \left\{\left(x, y : x + y < 3\right)\right\}$$

d) Find A, B and C if $A\left(x-1\right)^2 + B\left(x-1\right) + C \equiv 3x^2 + 5x + 8$ 3

e) Differentiate the following from first principles: 3

$$f(x) = 3x^2 - 2x$$

f) Given that $y = x^2 + 5x - 3$ and $9x - y - 7 = 0$. 4

(i) Find any points of intersection.

(ii) Prove that $9x - y - 7 = 0$ is a tangent to $y = x^2 + 5x - 3$

End of Question 3

Start a new booklet.

Question 4 (16 marks). **Marks**

a) Solve: $9\times 3^{x-1}=\frac{1}{27}$ 2

b) Find the exact solution to the equations: 4

(i) $\ln(3x-7)=5$

(ii) $3^x e^{7x+2}=15$

c) The functions *f* and *g* are defined by: 3

$$f(x)=e^{2x}+3$$

$$g(x)=\ln(x-1)$$

Find $f[g(x)]$ and state its range.

d) Bag A contains 2 black and 5 white balls. Bag B contains 4 black and 6 white balls. One bag is selected at random and 2 balls taken from it, without replacement. What is the probability that one ball is black and one ball is white? 3

e) (i) Show that the locus of the point P, which moves so that its distance from A(1,2) is always three times its distance from B(5,6), is a circle. 4

(ii) State its centre and radius.

End of Question 4

Start a new booklet.

Question 5 (16 marks). **Marks**

a) Sketch the derivative of $y = |x|$. 2

b) The curve C has the equation: 9

$$y = x^2\left(x-6\right)+\frac{4}{x};\ x>0\,.$$

The points P and Q lie on C and have x co-ordinates 1 and 2 respectively.

(i) Show that the length of PQ is $\sqrt{170}$.

(ii) Show that the tangents to C at P and Q represent the same line.

(iii) Find an equation of the normal to C at P giving your answer in general form.

c) If a car dealership sets the price of their cars at \$28000, they will sell 54 cars. Every time they drop the price \$1000, 2 more cars will be sold. What should the price of their cars be set at to maximise income? 5

End of Question 5.

End of Examination.

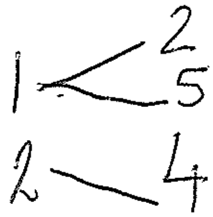
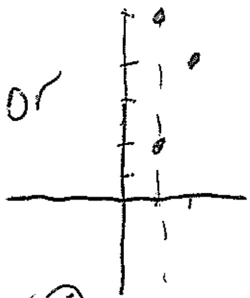
Solutions Nov 2011 YR 11 Assess Task 1. $\frac{}{16}$

1 (a) $86 = 3 \times 27 + 7$ NO! 1

$86 = 88$

(b) 11 1

(c) $5^x = 625$ 1

(d)  or 

NO 1

(e) $a = 2,\ b = -1,\ c = -3$

$\Delta = (-1)^2 - 4 \times 2 \times -3$

$= 1 + 24$

$= 25$

(i) real 1

(ii) distinct 1

(iii) rational 1

(f) (i) $f(-x) = (-x)^3 + 3(-x)^2 + 7(-x)$

$= -x^3 + 3x^2 - 7x$

$= -(x^3 - 3x^2 + 7x)$

$\neq f(x)$, no odd or even.

or $\neq -f(x)$ neither. 1

(ii) $f(-x) = \dfrac{2}{(-x)^3} = \dfrac{2}{-x^3} = -\dfrac{2}{x^3}$.

So not even $\neq f(x)$

but $f(-x) = -f(x)$ odd 1

(iii) $f(-x) = \dfrac{1}{(-x)^5 + (-x)^3} = \dfrac{1}{-x^5 - x^3} = \dfrac{-1}{x^5 + x^3}$ $\neq f(x)$ not even

$= -f(x)$

odd. 1

(g) $x^2 + 3px + p$

has equal roots.

$a = 1$ $\quad \Delta = b^2 - 4ac = 0$

$b = 3p$ $\quad 9p^2 - 4 \times 1 \times p = 0$

$c = p$

$$9p^2 - 4p = 0$$

$$p(9p - 4) = 0$$

$$p = 0 \text{ or } p = \frac{4}{9}$$

2

but $p \neq 0$ data so $p = \frac{4}{9}$

(h) P(Albert pass, Charlie pass, Byron fail)

$$= \frac{1}{2} \times \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{4}$$

$$= \frac{1}{8}$$

2

(i) $\log 5 + 2\log 8 - \frac{1}{2}\log 16 = \log x$

LHS $\quad \log 5 + \log 8^2 - \log 16^{\frac{1}{2}}$

$\log 5 + \log 64 - \log 4$

$\log\left(\frac{5 \times 64}{4}\right)$

$\log 80 = \log x \qquad x = 80$

2

2UNIT SOLUTIONS

QUESTION 2

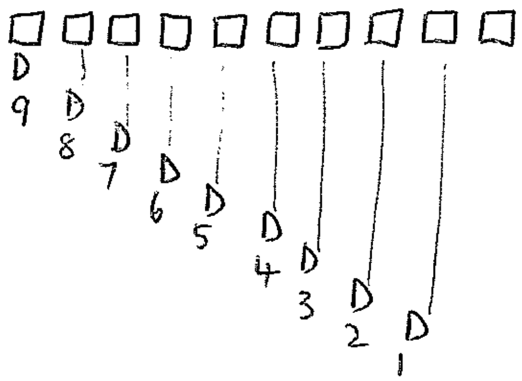
a) Defective $0{\cdot}006$ 2

Non-Defective $0{\cdot}994$

2 Defective means 8 Non-Defective.

ie. $0{\cdot}006 \times 0{\cdot}006 \times (0{\cdot}994)^8$

Also, how many different positions can defective ones be arranged, amongst the ten in the box



ie. $9+8+7+6+5+4+3+2+1=45$

$\therefore\ P(E) = 45 \times 0{\cdot}006 \times 0{\cdot}006 \times (0{\cdot}994)^8$

$=$

[OR $^{10}C_2 \times 0{\cdot}006^2 \times 0{\cdot}994^8$]

b) (i) $\boxed{5x^4 - 16x^3 + 4x}$ 1

(ii) $\boxed{21(3x+2)^6}$ 1

(iii) $2x^3 - 13 + x.6x^2$

$= 2x^3 - 13 + 6x^3$ 2

$= \boxed{8x^3 - 13}$

(iv) $\dfrac{(x-1).16x - (8x^2+6)}{(x-1)^2}$

$= \dfrac{16x^2 - 16x - 8x^2 - 6}{(x-1)^2}$

$= \boxed{\dfrac{8x^2 - 16x - 6}{(x-1)^2}}$

$= \dfrac{2(4x^2 - 8x - 3)}{(x-1)^2}$ 2

c) $y = x^2 + 2x$ 3

$\dfrac{dy}{dx} = 2x + 2$

but $m = 4$ as lines parallel

$\therefore\ 2x + 2 = 4$

$2x = 2$

$x = 1$

and so $y = 3$.

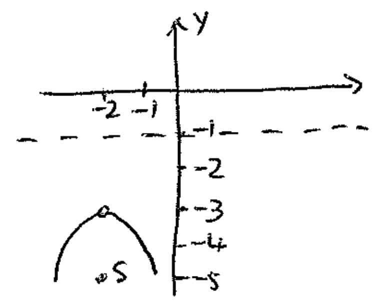
Equ'n of tang: $m=4$ $(1,3)$

$y - 3 = 4(x-1)$

$y - 3 = 4x - 4$

$\boxed{y = 4x - 1}$

d) 5



(i) Focal length $= 2$

(ii) Focus $(-2, -5)$

(iii) $x = -2$

(iv) $(x+2)^2 = -4a(y+3)$

$(x+2)^2 = -8(y+3)$

Marking Scale.

a) 0.006×0.006 1 mark.

0.06×0.06 OR the like $\frac{1}{2}$ mark

$0.006^2 \times 0.994^8$ $1\frac{1}{2}$ marks

$(0.994)^8$ 1 mark

b) (i) 1 mark (no half marks)

(ii) 1 mark (" " ")

$7(3x+2)^6 \cdot 3$ is OK.

(iii) One part of product rule correct – 1 mark
(no half marks)

(iv) One error – 1 mark.
(no half marks)

c) $\left.\begin{array}{l} 2x+2=4 \\ \text{OR } y' = 2x+2 \end{array}\right\}$. 1 mark

Point $(1,3)$ 1 mark

Equn $y = 4x - 1$ 1 mark.
(No half marks)

d) (i) $a = -2$ $\frac{1}{2}$ mark.

(ii) no half marks.

(iii) -2 $\frac{1}{2}$ mark

$y = -2$ $\frac{1}{2}$ mark.

(iv) Leaving out the '–' sign
– (minus 1 mark)

Using focus instead of vertex
– (minus 1 mark)

1 2 3 3 3 4 Solution to Question (3)

(a) $\lim_{x \to 2} \frac{2(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

[1] $= \boxed{8.}$ ———— *

(b) $5^x = 7$

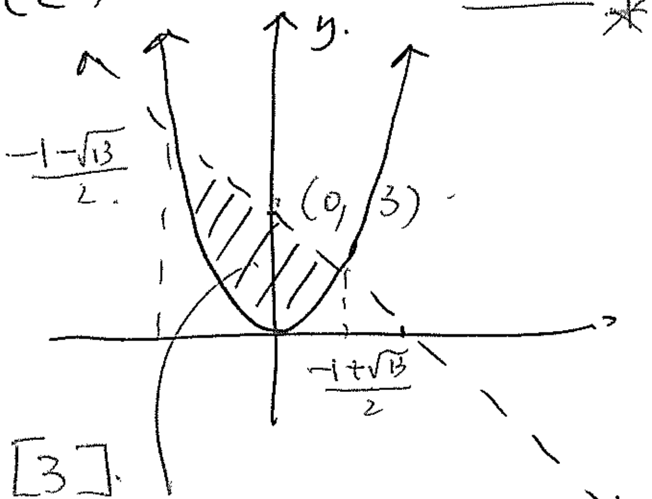
$x \log_{10} 5 = \log_{10} 7$

$\therefore x = \frac{\log_{10} 7}{\log_{10} 5}$

[2] $\boxed{x = 1.21}$

———— *

(c)



[3]. *

(d) $A(x-1)^2 + B(x-1) + C$

$\equiv 3x^2 + 5x + 8$

$x = 1$, $\boxed{C = 16}$

Coeff of x^2 $\boxed{A = 3}$

$x = 0$

$8 = A - B + C$ [3]

$8 = 3 - B + 16$

$\therefore -B = -11$

$\therefore \boxed{B = 11}$

(e) $f(x+h)$

$= 3(x+h)^2 - 2(x+h)$

$= 3[x^2 + 2xh + h^2]$

$- 2x - 2h$

$= 3x^2 + 6xh + 3h^2$

$- 2x - 2h$

$\therefore$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{6x\cancel{h} + 3h^2 - 2h}{\cancel{h}}$

$= 6x - 2.$ [3].

(f). $9x - 7 = x^2 + 5x - 3$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

(i) $\therefore x = 2.$ $y = 11$

$(2, 11)$ is pt of intersection.

$x^2 - 4x + 4 = 0$

(ii) $\Delta = 0$ [4]

$\therefore y = 9x - 7$

is a tgt. to

$y = x^2 + 5x - 3$

Since $\exists$ only a solution

QUESTION 4

(a) $3^2 \times 3^{x-1} = 3^{-3}$

$x + 1 = -3$

$x = -4$

(b) (i) $3x - 7 = e^5$

$x = \frac{1}{3}(e^5 + 7)$

(ii) $\ln(3^x e^{7x+2}) = \ln 15$

$\ln 3^x + \ln e^{7x+2} = \ln 15$

$x \log_e 3 + 7x + 2 = \log_e$

$x(\log_e 3 + 7) = \ln 15 - 2$

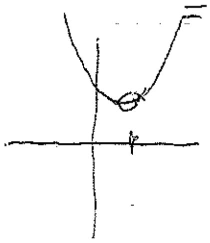
$x = \dfrac{\ln 15 - 2}{\ln 3 + 7}$

(c) $g(x) = \ln(x-1)$

$f[g(x)] = e^{2\ln(x-1)} + 3$

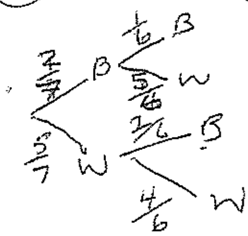
$= e^{\log_e (x-1)^2} + 3$

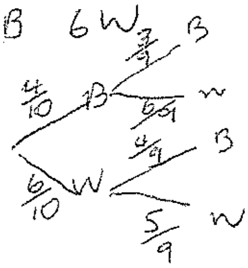
$= (x-1)^2 + 3$



Domain all real x.
Range $y > 3$.

(d) (A) 2B 5W (B) 4B 6W

(A) 

B 

$\frac{1}{2}\left(\frac{2}{7} \times \frac{5}{6} + \frac{5}{7} \times \frac{2}{6}\right)$ $\frac{1}{2}\left(\frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{4}{9}\right)$

$\frac{5}{21} + \frac{4}{15}$ $\frac{53}{105}$

or

$\frac{1}{2} \times \dfrac{{}^2C_1 \times {}^5C_1}{{}^7C_2} + \frac{1}{2} \times \dfrac{{}^4C_1 \times {}^6C_1}{{}^{10}C_2}$

$= \frac{53}{105}$

(e) $\sqrt{(x-1)^2 + (y-2)^2} = 3\sqrt{(x-5)^2 + (y-6)^2}$

$x^2 - 2x + 1 + y^2 - 4y + 4$

$= 9x^2 - 90x + 225 + 9y^2 - 108y + 324$

$8x^2 - 88x + 8y^2 - 104y + 544 = 0$

$x^2 - 11x + \left(\frac{11}{2}\right)^2 + y^2 - 13y + \left(\frac{13}{2}\right)^2 = -68 + 72\frac{1}{2}$

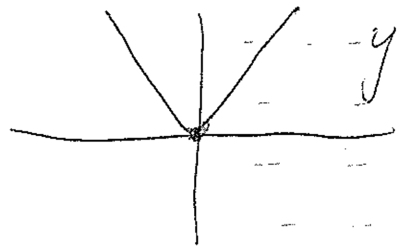
$\left(x - \frac{11}{2}\right)^2 + \left(y - \frac{13}{2}\right)^2 = \frac{9}{2}$

Circle $C\left(\frac{11}{2}, \frac{13}{2}\right)$ r $\frac{3\sqrt{2}}{2}$

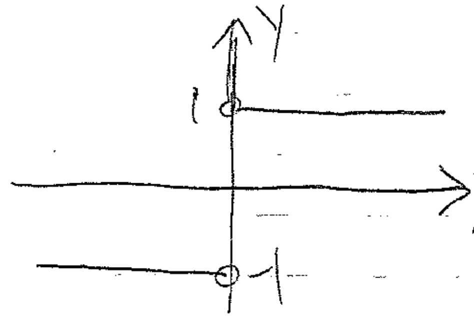
Q5. 11 Maths

5.

(a) $y = |x|$



$y = f(x)$



$y = f'(x)$

2

(b) $y = x^2(x-6) + \frac{4}{x}$; $x > 0$

P, Q lie on curve $P(1, y_p)$

$Q(2, y_q)$

(i) Show $\overline{PQ} = \sqrt{170}$.

When $x = 1$, $y = 1(-5) + \frac{4}{1} = -1$

$\Rightarrow \boxed{P(1, -1)}$

When $x = 2$, $y = 4(-4) + \frac{4}{2} = -14$

$\Rightarrow \boxed{Q(2, -14)}$

$d_{PQ} = \sqrt{(2-1)^2 + (-14+1)^2}$

$= \sqrt{1 + 169} = \sqrt{170}$.

3

(ii) $y' = \cancel{x^2 \cdot 1 + (x-6)2x} - \frac{4}{x^2}$

$= 3x^2 - 12x - 4/x^2$

Point P $y'(1) = 1 + -10 - 4 = -13$

Point Q $y'(2) = 4 - 16 - 1 = -13$

(ii) (cont) Then eqn of tangent at $P(1,-1)$ $m=-13$

is $y+1=-13(x-1)$

$y+1=-13x+13$

$y=-13x+12$ (1) ✓

(4)

Eqn of tangent at $Q(2,-14)$ and $m=-13$

$y+14=-13(x-2)$

$=-13x+26$

$y=-13x+12$ (2) ✓

$m_1=m_2$ $\therefore$ parallel + same line

(iii) Normal at P $\Rightarrow$ $m=\frac{1}{13}$ ✓

(2)

$y+1=\frac{1}{13}(x-1)$

$13y+13=x-1$

$-x+13y+14=0$ #. ✓

(c)

Price	n (no sold)
28000	54
28000 − 1000	54 + 2
28000 − 2×1000	54 + 2×2

$x=$ no of \$1000 reductions

Income = Price × n

$I=(28000-1000x)(54+2x)$ ✓

$$I = 1512000 + 56000x - 54000x - 2000x^2$$

$$\boxed{I = 1512000 + 2000x - 2000x^2}$$

$$I' = 2000 - 4000x$$

For tp's $I' = 0 \Rightarrow -4000x = -2000$

$$x = \tfrac{1}{2}$$

$$I'' = -4000 \Rightarrow \text{max at } x = \tfrac{1}{2}.$$

or T.P. at $x = \frac{-b}{2a}$. (quadratic)

$$= \frac{-2000}{-4000}$$

$x = \frac{1}{2}$ and $a < 0$ $\therefore$ max.

When $x = \frac{1}{2}$, $I = \$1513500$.

$\therefore$ Price should be set at

$$\$28000 - 1000 \times \tfrac{1}{2}$$

$$= \$27500$$